St Joseph's College (Autonomous)

Bengaluru-560027

Department of Mathematics

Syllabus for Postgraduate Program

Re-accredited with **'A++' Grade and 3.794 CGPA** by NAAC

Recognized by UGC as College of Excellence

For Batch 2018-2021

SUMMARY OF CREDITS

Department of Mathematics (PG)
(2018-2021)

Semester	Theory/ Practical	Course Code	Cousre Title	Number of hours/week	Number of credits	Max marks for end sem	Duration of end sem
1	Theory	MT7118	Algebra I	4	4	70	2.5 hours
	Theory	MT7218	Real Analysis	4	4	70	2.5 hours
	Theory	MT7318	Linear Algebra	4	4	70	2.5 hours
	Theory	MT7418	Ordinary Differential Equations	4	4	70	2.5 hours
	Theory	MT7518	Discrete Mathematics	4	4	70	2.5 hours
	Practical	MT7P1	Practicals I	3	2	35	3 hours
	Practical	MT7P2	Practicals II	3	2	35	3 hours
2	Theory	MT8118	Algebra II	4	4	70	2.5 hours
	Theory	MT8218	Complex Analysis	4	4	70	2.5 hours
	Theory	MT8318	Theory of Numbers	4	4	70	2.5 hours
	Theory	MT8418	Partial Differential Equations	4	4	70	2.5 hours
	Theory	MT8518	Topology	4	4	70	2.5 hours
	Practical	MT8P1	Practicals I	3	2	35	3 hours
	Practical	MT8P2	Practicals II	3	2	35	3 hours
3	Theory	MT9118	Functional Analysis	4	4	70	2.5 hours
	Theory	MT9218	Classical and Continuum Mechanics	4	4	70	2.5 hours
	Theory	MTDE9X18	Elective I	4	4	70	2.5 hours
	Theory	MTDE9X18	Ecelctive II	4	4	70	2.5 hours
	Project	MT9R1	Introduction to Mathematical Research	2	2	50	NA
4	Theory	MT0118	Measure and Integration	4	4	70	2.5 hours
	Theory	MTDE0X18	Elective I	4	4	70	2.5 hours
	Theory	MTDE0X18	Elective II	4	4	70	2.5 hours
	Theory	MTDE0X18	Elective III	4	4	70	2.5 hours
	Project	MT9R2	Project	2	6	100	NA

Total Number of Credits = 88

FIRST SEMESTER

1. ALGEBRA - MT7118

(4 hours per week)

GROUP THEORY

Permutation (or Symmetric) groups: Cycles. Every permutation of a finite set can be written as a product of disjoint cycles. Disjoint cycles commute. Order of a permutation. Even or Odd permutation. alternating group. (3 hours)

Dihedral groups: Dihedral Groups. Generators and Relations. (3 hours)

Isomorphisms: Isomorphism theorems and its related problems. (2 hours)

Group action on a set: Group action and permutation representation. Kernel. Orbit and Stabilizer. Group action defines an equivalence class on the group. Orbit-Stabilizer Theorem. Groups acting on itself and Cayley,s Theorem. Class Equation. Application of class equation. Number of conjugacy classes of the permutation group. (8 hours)

Automorphisms: Automorphism. Inner automorphism. (3 hours)

Sylow's Theorem: p-subgroup. Sylow- p subgroup. Sylow's Theorem. Application of Sylow's Theorem. Simplicity of Alternating Group. (8 hours)

RING THEORY

Polynomials Rings: $D[x]$ is an integral domain if D is an integral domain. Division algorithm. Remainder Theorem. Factor Theorem. Polynomials of degree n has at most n zeros counting multiplicity. PID. $\mathbb{F}$ is a field implies $\mathbb{F}[x]$ is a PID. Irreducible and Reducible Polynomial. Different Reducibility Test. Primitive polynomial. Content of a polynomial. Gauss lemma. Eisenstein criteria. Unique Factorization Domains. $\mathbb{Z}[x]$ is an UFD. (8 hours)

FIELD THEORY

Field Theory: Basic theory of field extensions. Algebraic extensions. Classical straightedge and Compass construction. Splitting field. Algebraic closure. Separable and inseparable extensions. Cyclotomic polynomials. (12 hours)

GALOIS THEORY

Galois Theory: Elements of Galois Theory. Fixed fields. Normal extension. Galois groups over the rationals. (13 hours)

TEXT BOOKS:

1) D. S. Dummit and R. M. Foote. Abstract Algebra. Wiley. 2003

2) J. A. Gallian. Contemporary Abstract Algebra. 4th Edition. Narosa Publishing. 2011

REFERENCE BOOKS:

1) C. S. Musili. Rings and Modules . 2nd Revised Edition. Narosa Publishing House. 1994

2) I. N. Herstein. Topics in Algebra. 2nd Edition. Wiley. 1975

3) I. S. Luthar and I. B. S. Passi. Algebra Volume-I Groups. Narosa Publishing House. 2013

4) I. S. Luthar and I. B. S. Passi. Algebra Volume-II Rings. Narosa Publishing House. 2012

5) J. B. Fraleigh. A first course in Abstract Algebra. 7th Edition Pearson Education India. 2002
6) M. Artin . Algebra. 2nd Edition Pearson Education India. 2017
7) S. K. Mapa. Higher Algebra Abstract and Linear. Sarat Book House. 1972
8) S. Lang. Algebra. 3rd Edition. Springer. 2002
9) S. Lang. Undergraduate Algebra. Springer. 1987

2. REAL ANALYSIS - MT7218

(4 hours per week)

Countability: Finite, Infinite, Denumerable, Countable, Uncountable sets. Cardinality of sets. Equivalent sets. The set of real numbers is uncountable. Denumerable union of denumerable set is again denumerable. The set of rational numbers is denumerable. (4 hours)

Riemann Integration: Partition of a closed bounded interval. Riemann integrable functions. Darboux Integrable functions. Riemann Integrable is equivalent to Darboux Integrable. Necessary and sufficient condition of a Riemann Integrable function. Continuous function is Riemann Integrable. Sandwich theorem. Riemann-Lebesgue Theorem. Monotonic function is Riemann integrable. Bounded function with finitely many discontinuities is Riemann Integrable. Composition of Riemann Integrable function with a continuous function is continuous. Algebraic properties of Riemann Integration. Change of variable. Fundamental theorem of calculus. Integration by parts. (15 hours)

Sequence and Series of Functions: Pointwise convergence of sequence of functions. Different examples. Uniform convergence. Necessary sufficient conditions for uniform convergence. Uniform convergence and continuity. Uniform convergence and integration. Uniform convergence and differentiation. Stone-weierstrass Theorem. (10 hours)

METRIC SPACES

Introduction to Metric Spaces: Definition. Basic Example (with emphasis on set of all real-valued continuous bounded function on $[0, 1]$). Distance between a point and a subset of metric space. Diameter of a subset of metric space. (4 hours)

Topology on Metric Spaces: Open Sphere. Open Set. Examples of different open sets. any open set is a union of open sphere. arbitrary union of open set is an open set; finite intersection of open set is an open set. Interior points. any set A is open iff all the points of A are interior points of A. (3 hours)

Limit Point. Closed Set. A subset is closed iff its complement is open. Closed sphere. Closed spheres are closed sets. Arbitrary intersection of closed set is closed. Finite union of closed set is closed. Cantor Set (Optional). Boundary point. A subset is closed iff it contains its boundary points. (3 hours)

Sequences in Metric Spaces: Convergence of a sequence in Metric Space. Cauchy Sequence. Complete Metric Space. Examples of Complete Metric Spaces. A subset of a complete metric space is closed iff it is complete. Cantor-Intersection Theorem. Baire-Category Theorem. (8 hours)

Continuous Mapping Between two Metric Spaces: Definition of continuity between two metric spaces. A mapping is continuous iff convergent sequence is mapped to convergent sequence. Inverse of an open set via a continuous mapping is continuous. (3 hours)

FUNCTIONS OF SEVERAL VARIABLES

Functions of several variables. Continuity and Differentiation of vector-valued functions. linear transformation of R^k, properties and invertibility. Directional Derivative. Chain rule. Partial Derivative. Hessain matrix. The Inverse Functions Theorem and its illustrations with examples. The Implicit Functions Theorem and illustrations and examples. The Rank theorem illustration and examples. (10 hours)

TEXT BOOKS:

1) C. C. Pugh. Real Mathematical Analysis. 2nd Edition. UTM Springer. 2002
2) G. F. Simmons. Introduction to topology and Modern Analysis. 1st Edition. McGraw-Hill Education. 1963
3) J. Munkres. Topology. 2nd Edition. Pearson Education India. 2000
4) D. R. Sherbert and G. Bartle. Introduction to Real Analysis. 4th Edition. Wiley. 2014

REFERENCE BOOKS:

1) W. Rudin. Principles of Mathematical Analysis. 3rd Edition. McGraw-Hill Education. 1976
2) J. M. Howie. Real Analysis. Springer India. 2001
3) S. K. Berberian. A first course in Real Analysis. Springer India. 1994
4) S. K. Mapa. Introduction to Real Analysis. 7th Edition. Sarat Book House. 2013
5) S. Kumaresan. Topology of Metric Spaces. 2nd Edition. Narosa. 2005
6) S. R. Ghorpade and B. V. Limaye. A course in Calculus and Real Analysis. 1st Edition. Springer. 2006
7) S. Shirali and H. L. Basudeva. Metric Spaces. Springer. 2006
8) T. Tao. Analysis-I. 3rd Edition. Hindustan Book Agency. 2016

3. LINEAR ALGEBRA - MT7318

(4 hours per week)

Recapitulation: Vector spaces and Linear Transformations. (2 hours)

Determinants: Determinants of Matrix. Properties of Determinant. Annihilating polynomials and minimal polynomials. (9 hours)

Diagonalization: Eigen Value and Eigen Vectors. Diagonalizability. Invariant Subspaces.

Cayley-Hamilton Theorem. (10 hours)

Inner Product Spaces: Inner Product and Norms. The Gram-Schmidt Orthogonalization Process. Orthogonal Complement. Adjoint of a Linear Operator. Self-adjoint and Normal Operators. Unitary and Orthogonal Matrices and Operator. (15 hours)

Positive Definiteness: Positive definite matrices. Maxima, minima and saddle points. Test for positive definiteness. Singular value decomposition and its application. (8 hours)

Quadratic forms: Bilinear form and Quadratic form. (8 hours)

Canonical Forms: Generalized Eigenvector. Jordan Canonical Form. The minimal Polynomial (8 hours)

TEXT BOOKS:

1) A.J.Insel. L.E.Spence and S.Friedberg: Linear Algebra. 4th Edition. Pearson Education. 2003
2) A.R. Rao and P.Bhimasankaram : Linear Algebra. Hindustan Book Agency. Second Edition. 19 TRIM Series. 2010
3) S. K. Mapa : Higher Algebra Abstract and Linear. revised 9th Edition. Sarath Book House. 2003

REFERENCE BOOKS:

1) C. W. Curtis : Linear Algebra an Introductory Approach. 4th Edition. Springer. 1984
2) D. C. Lay : Linear Algebra and its Application. 3rd Edition. Pearson Education India. 2009
3) G. Strang : Linear Algebra and Its Application. 4th Edition. Cengage Learning. 2006
4) K. Hoffman and R. Kunze :Linear Algebra. 2nd Edition. Pearson Hall India Ltd. 1978
5) S. Lang :Linear Algebra. 3rd Edition . 11th Printing. Springer. 2004
6) S. Kumaresan : Linear Algebra-A geometric approach. Prentice Hall India Private Limited. 2000.

4. ORDINARY DIFFERENTIAL EQUATIONS - MT7418

(4 hours per week)

Linear differential equations of n^{th} order. Fundamental sets of solutions. Wronskian-Abel's identity. Theorems on linear dependence of solutions. Adjoint, Self-adjoint linear operator. Green's formula. Adjoint equations. The n^{th} order non-homogeneous linear equations. Variation of parameters. Zeros of solutions. Comparison and separation theorems. (15 hours)

Fundamental existence and uniqueness theorem. Dependence of solutions on initial conditions. Existence and uniqueness theorem for higher order and system of differential equations. Eigenvalue problems. Sturm-Liouville problems. Orthogonality of eigen functions . Eigen function expansion in a series of orthonormal functions. Green's function method. (15 hours)

Power series solution of linear differential equations. Ordinary and singular points of differential equations. Classification into regular and irregular singular points. Series solution about an ordinary point and a regular singular point. Frobenius method. Hermite. Laguerre. Chebyshev and Gauss Hypergeometric equations and their general solutions. Generating function. Recurrence relations. Rodrigue's formula. Orthogonality properties. Behavior of solution at irregular singular points and the point at infinity. (15 hours)

Linear system of homogeneous and non-homogeneous equations (matrix method). Linear and Non-linear autonomous system of equations. Phase plane - Critical points. Stability. Lyapunov direct method. Limit cycle and periodic solution. Bifurcation of plane autonomous systems. (15 hours)

TEXT BOOKS:

1) G.F. Simmons. Differential Equations. Tata McGraw Hill Edition. New Delhi. 1974.
2) M.S.P. Eastham: Theory of ordinary differential equations. Van Nostrand. London. 1970.
3) S.L. Ross: Differential equations. John Wiley and Sons. New York. 3rd edition. 1984.

REFERENCE BOOKS:

1) A.C.King. J.Billingham and S.R.Otto. Differential equations. Cambridge University Press. 2006.
2) E.A. Coddington and N. Levinson. Theory of ordinary differential equations. McGraw Hill. 1955.
3) E.D. Rainville and P.E. Bedient. Elementary Differential Equations. McGraw Hill. New York. 1969.

5. **DISCRETE MATHEMATICS - MT7518**

(4 hours per week)

Introduction to logic. Methods of Proof- rules of inference, valid arguments, rules of inference for quantified statements. Methods of proving theorems- direct proofs, indirect proofs, proof by contradiction, proof by cases. Proofs of equivalence. (5 hours)

Basic counting principles. The product rule and the sum rule. Examples to illustrate sum and product rule. The inclusion-exclusion principle and examples. The Pigeonhole Principle and examples. Simple arrangements and selections. Arrangement and selections with repetitions. Distributions. Binomial coefficients. (8 hours)

Recurrence relations. Modeling with recurrence relations with examples of Fibonacci numbers. The tower of Hanoi problem. Divide-and-conquer relations with examples (no theorems). Generating function definition with examples. List of generating functions. Difference equations. (9 hours)

Definition and types of relations. Representing relations using matrices and digraphs. Closures of relations. Paths in digraphs. Transitive closures. Warshall's Algorithm. Partial

Orderings. Hasse diagrams. maximal and Minimal elements. Lattices. (8 hours)

Introduction to graph theory. Types of graphs. Basic terminologies . Subgraphs. Representing graphs as incidence matrix and adjacency matrix. Graph isomorphism. Connectedness in simple graphs. Paths and cycles in graphs and digraphs. Distance in graphs- eccentricity, radius, diameter, center, periphery. Weighted graphs Dijkstra's algorithm to find the shortest distance paths in graphs and digraphs. (9 hours)

Euler and Hamiltonian Paths. Necessary and sufficient conditions for Euler circuits and paths in simple. undirected graphs. Hamiltonicity: noting the complexity of hamiltonicity. Traveling Salesman's Problem. Nearest neighbour method. (7 hours)

Planarity. Plane and planar graphs. Euler Identity. Non planar graphs. Maximal planar graphs. Outer planar graphs. Maximal outer planar graphs. Characterization of planar graphs. Geometric dual. crossing number. Kuratowski's theorem (statement only). Weighted graphs. Vertex connectivity. Edge connectivity. Covering. Independence. (7 hours)

Trees. Rooted trees. Binary trees. Trees as models. Properties of trees. Minimum spanning trees. Minimum spanning trees. Prims and Kruskul Algorithms. (7 hours)

TEXT BOOKS:

1) C. L. Liu: Elements of Discrete Mathematics. Tata McGraw-Hill. 2000.
2) F.Harary: Graph Theory. Addison-Wesley. 1969.
3) K.Rosen. WCB McGraw-Hill. 6th edition. 2004.

REFERENCE BOOKS:

1) C.T.Leondes. Control and Dynamic systems. Academic Press-2006.
2) J.P. Tremblay and R.P. Manohar . Discrete Mathematical Structures with applications to computer science. McGraw Hill. 1975.
3) J.H.Van Lint and R.M. Wilson. A course on combinatorics. Cambridge University Press. 2006.

PRACTICALS I - MT7P1

Discrete Mathematics Practicals using Python

(3 hours per week per batch)

LIST OF PROBLEMS

1. Logical operators: And. Or. Not. Nand. Nor. Xor. Implies. Equivalent . Unequal.
2. Finding CNF and DNF.
3. Solving recurrence/difference relations(with and without boundary conditions).
4. Finding a generating function (given a sequence).
5. Digraph representations (plotting) of relations with their properties.
6. Hasse diagrams.
7. Lattice properties including the extremal values.

8. Graph Isomorphism (using algorithms like NAUTY).
9. Counting paths and their lengths (like Dijkstra's Algorithm)
10. Constructing Eulerian Cycles.
11. Traveling Salesman Problem.
12. Determining whether given adjacency matrix represents a tree.
13. Determining Minimum spanning trees (Prim's/ Kruskal's algorithms).

PRACTICALS II - MT7P2

Linear Algebra Practicals using Python

(3 hours per week per batch)

LIST OF PROBLEMS

1. Linear dependence, independence. Linear combinations. Change of basis.
2. Linear transformation to matrix conversion and vice versa.
3. Change of basis matrix
4. Finding Eigen valuse and their multiplicities as roots of $\det(A - \lambda I) = 0$
5. Calculation of linearly independent eigen vectors.
6. Calculation of Eigenvalues and eigen vectors for Symmetric matrix.
7. Orthogonal and orthonormal sets.
8. Eigen values and orthonormal eigen vectors.
9. Gram-Schmidt Orthogonalization of column vectors.
10. Diagonalization.
11. Triangularization.
12. Singular Value Decomposition.

SECOND SEMESTER

1. ALGEBRA II - MT8118

(4 hours per week)

Fundamental Theorem of Finite Abelian Groups: External Direct Product. Structure theorem for finite Abelian Group. (6 hours)

Free Groups: Existence and Universal Properties of Free groups. Examples of groups specified by Generators and Relations. (3 hours)

Nilpotent and Solvable Groups: Nilpotent Groups. Solvable groups. A finite group is a direct product of its Sylow groups. Commutators and Lower Central Series. Criterion for a nilpotent group. Solvable groups and Derived Series. Criterion for a solvable group. (8 hours)

Simple Groups: Simple Groups. Composition Series. Jordan-Holder Theorem. The Holder Program. (6 hours)

Ring Theory: Euclidean Domain. PID. UFD. Every Euclidean Domain is an PID. Prime and irreducible elements an integral domain. In a PID any non-zero element is prime iff it is irreducible. In a UFD a non-zero element is prime iff it is irreducible. Every PID is a UFD. Z is a UFD. Gaussian Integer. Fermat's Theorem on sum of squares. (10 hours)

MODULE THEORY

Introduction to Modules: Modules and Module Homomorphism. Examples of Modules. Submodules. Algebra. Quotient Modules. Operations on Submodules. Generators of a Modules. Direct Sums and Product of Modules. Finitely Generated Module. Nakayama's Lemma . Exact Sequence of Modules. Snake's Lemma. (15 hours)

Free Modules. Universal Property of Free Modules. (1 Hour)

Tensor Product of Modules. Split Exact Sequence. Projective Modules. Injective Modules. (13 hours)

TEXT BOOKS:

1) D. S. Dummit and R. M. Foote. Abstract Algebra. Wiley. 2003
2) J. A. Gallian. Contemporary Abstract Algebra. . 4th Edition. Narosa Publishing. 2011
3) N.S. Gopalakrishnan. Commutative Algebra. 2nd Edition. Orient Blackswan. 2015
4) M.F. Atiyah and I.G. Macdonald. Introduction to Commutative Algebra. Addison-Wesley Series In Mathematics. Avalon Publishing. 1994

REFERENCE BOOKS:

1) C. S. Musili. Rings and Modules . 2nd Revised Edition. Narosa Publishing House. 1994
2) I. N. Herstein. Topics in Algebra. 2nd Edition. Wiley. 1975
3) I. S. Luthar and I. B. S. Passi. Algebra Volume-I Groups. Narosa Publishing House. 2013
4) I. S. Luthar and I. B. S. Passi. Algebra Volume-II Rings. Narosa Publishing House. 2012

5) I. S. Luthar and I. B. S. Passi. Algebra Volume-III Modules. Narosa Publishing House. 2013

6) J. B. Fraleigh. A first course in Abstract Algebra. 7th Edition Pearson Education India. 2002

7) M. Artin . Algebra. 2nd Edition Pearson Education India. 2017

8) S. Lang. Algebra. 3rd Edition. Springer. 2002

9) S. Lang. Undergraduate Algebra. Springer. 1987

2. COMPLEX ANALYSIS - MT8218

(4 hours per week)

Analytic functions. Harmonic conjugates. Elementary functions. Mobius Transformation. Conformal mappings. Cauchy's Theorem and Integral formula. Morera's Theorem. Cauchy's Theorem for triangle and rectangle. Cauchy's Theorem in a disk. Zeros of Analytic function. The index of a closed curve. counting of zeros. Principle of analytic Continuation. Liouville's Theorem. Fundamental theorem of algebra. (8 hours)

Series. Uniform convergence. Power series. Radius of convergences. Power series representation of Analytic function. Relation between Power series and Analytic function. Taylor's series. Laurent's series. (12 hours)

Rational Functions. Singularities. Poles. Classification of Singularities. Characterization of removable Singularities. poles. Behavior of an Analytic function at an essential singular point. (12 hours)

Entire and Meromorphic functions. The Residue Theorem. Evaluation of Definite integrals. Argument principle. Rouche's Theorem. Schwarz lemma. Open mapping theorem and Maximum modulus theorem and its applications. Convex functions. Hadamard's Three circle theorem. (16 hours)

Phragmen-Lindelof theorem. The Riemann mapping theorem. Weierstrass factorization theorem. Harmonic functions. Mean Value theorem. Poisson'ś formula. Poisson's Integral formula. Jensen's formula. Poisson's-Jensen's formula. (12 hours)

TEXT BOOKS:

1) J. B. Conway. Functions of one complex variable. Narosa. 1987.

2) L.V. Ahlfors. Complex Analysis. McGraw Hill. 1986.

3) T. W. Gamelin. Complex Analysis. Springer-Verlag. 2006

REFERENCE BOOKS:

1) R. Nevanlinna. Analytic functions. Springer. 1970.

2) E. Hille. Analytic Theory. Vol-I. Ginn. 1959.

3. THEORY OF NUMBERS - MT8318

(4 hours per week)

Multiplicative and completely multiplicative functions. Euler Toteint function. Möbius and Mangoldt function. Dirichlet product and the group of arithmetical function. Generalised convolution. Formal power series. Bell series. (18 hours)

Residue Classes and complete Residue Classes. Linear Congruences an Euler-Fermat Theorem. General Polynomial congruences and Lagrange Theorem. Wilson's Theorem. Chinese Reminder Theorem. Fundamental Theorem on Polynomial Congruences with prime power moduli. Quadratic Residue and Gauss's Law of Quadratic Reciprocity. (Both for Legendre and Jacobi symbols) Primitive roots and their existence for moduli $m = 1,\ 2,\ 4,\ p\alpha, 2p\alpha$. (21 hours)

Partition: partition of a positive integer. Graphical representation. Conjugate. Generating functions. A theorem of Jacobi. Theorem 353 and 354. Applications of theorem 353. Congruence properties of $P(n)$. Two theorems of Euler. Rogers-Ramanujan Identities (portion to be covered as per Chapter-XIX of "An Introduction to the Theory of Numbers" written by G. H. Hardy and E. M. Wright.). (21 hours)

TEXT BOOKS:

1) Tom Apostl. Introduction to Analytic Number Theory. Springer 2010.
2) André Wiel. Number theory for beginners. Springer

REFERENCE BOOKS:

1) G H Hardy, E M Wright. An Introduction to the Theory of Numbers. Oxford University Press.
2) André Wiel. Basic Number Thery. Springer-Verlag
3) William Stein. Elementary Number Theory: Primes, Congruences, and Secrets: A Computational Approach. Springer 2009.

4. PARTIAL DIFFERENTIAL EQUATIONS - MT8418

(4 hours per week)

First Order Partial Differential Equations: Basic definitions. Origin of Partial Differential Equations. Classification. Geometrical interpretation. The Cauchy problem. The method of characteristics for Semi linear, quasi linear and Nonlinear equations. Complete integrals. Examples of equations to analytical dynamics. Discontinuous solution and shock-waves. (14 hours)

Second Order Partial Differential Equations: Definitions of Linear and Non-Linear equations. Linear Superposition principle. Classification of second-order linear partial differential equations into hyperbolic. parabolic and elliptic PDEs. Reduction to canonical forms. Solution of linear Homogeneous and non-homogeneous with constant coefficients. variable coefficients. Monge's method. (14 hours)

Wave equation: Solution by the method of separation of variables and integral transforms The Cauchy problem. Wave equation in cylindrical and spherical polar coordinates. (8 hours)

Laplace equation: Solution by the method of separation of variables and transforms. Dirichlet's. Neumann's and Churchill's problems. Dirichlet's problem for a rectangle. Half plane and circle. Solution of Laplace equation in cylindrical and spherical polar coordinates. (8 hours)

Diffusion equation: Fundamental solution by the method of variables and integral transforms. Duhamel's principle. Solution of the equation in cylindrical and spherical polar coordinates. (8 hours)

Solution of boundary value problems: Green's function method for Hyperbolic. Parabolic and Elliptic equations. (8 hours)

TEXT BOOKS:

1) N. Sneddon. Elements of PDE's. McGraw Hill Book company Inc. 2006
2) L. Debnath. Nonlinear PDE's for Scientists and Engineers. Birkhauser. Boston. 2007
3) F. John. Partial differential equations. Springer. 1971.

REFERENCE BOOKS:

1) F. Treves. Basic linear partial differential equations. Academic Press. 1975.
2) M. G. Smith. Introduction to the theory of partial differential equation. Van Nostrand. 1967
3) K .S. Rao. Partial Differential Equations. Prentice Hall India. 2006.

5. TOPOLOGY - MT8518

(4 hours per week)

Introduction to Topology : Definition and examples of topological spaces. Basis for a topology. Product Topology. Subspace Topology. Neighborhoods and Limit points. Closed Sets. Limit points. Closure. Interior and Boundary of a set. Hausdorff Space. (12 hours)
(Excluding the concept of finer and coarser, order topology, box topology)

Continuous Functions: Definition of continuous function. Equivalent definitions of continuity. and Homeomorphism. Pasting lemma. Maps into Product Spaces. (8 hours)
Metric topology. Sequence Lemma. Quotient Topology. (6 hours)

Connected spaces: Definition and examples. Union of connected set having a point in common is connected. Image of connected space is connected. Cartesian product of connected space is connected. path connected spaces. example of a topological space which is connected but not path connected (topologist's sine curve). Components and path components forms an equivalence relation. (11 hours)

Compactness: Definition and Examples of Compact Spaces. Closed subspace of compact space is compact. Compact subspace of a Hausdorff space is closed. Image of compact set is compact under a continuous map. The product of finitely many compact space is compact. Compactness and "finite intersection property". Lebesgue number lemma. Uniform continuity and compactness. (11 hours)

Separation Axioms: First countable and Second Countable topological space. Hausdorff Space. Regular Space. Normal Space. Necessary and Sufficient condition for Regular and Normal Spaces. Subspace of regular is regular. subspace of normal is normal. Urysohn's Lemma. Urysohn Metrization theorem. Tietze Extension Theorem. Tychnoff Theorem. (12 hours)

TEXT BOOKS:

1) G. F. Simmons. Introduction to Topology and Modern Analysis. Tata McGraw-Hill Education. 1963
2) J. Munkres. Topology. Pearson Education India. 2nd Edition. 2007

REFERENCE BOOKS:

1) J L. Kelley. General Topology. Van Nostrand. Princeton. 1955
2) J. B. Conway. A course in point set topology. UTM Series. Springer. 2013
3) K. D. Joshi. Topology. New Age International Private limited. 1983
4) M. A. Armstrong. Basic Topology. Springer India .1983.

PRACTICALS I - MT8P1

PDE Practicals using Python

(3 hours per week per batch)

LIST OF PROBLEMS

1. Obtaining partial derivative of some standard functions.
2. Classification of 2nd order PDE's s into parabolic. elliptic and hyperbolic PDE
3. Obtaining the solution of wave equation by Fourier decomposition method (Separation of variables).
4. Obtaining the solution of wave equation by Fourier transforms.
5. Obtaining the solution of Laplace equation by Fourier decomposition method (Separation of variables).
6. Obtaining the solution of Laplace equation by Fourier transforms.
7. Obtaining the solution of Heat equation by Fourier decomposition method (Separation of variables).
8. Obtaining the solution of Heat equation by Fourier transforms.
9. Implementing the green's function method for hyperbolic PDE.

10. Plotting of the double Fourier series solutions for wave equation and discussing about convergence.

PRACTICALS II - MT8P2

Theory of Numbers Practicals using Python

(3 hours per week per batch)

LIST OF PROBLEMS

1. Chinese Remainder Theorem
2. Power series
3. Residue Class and Complete Residue Class
4. Diophantine Equation
5. Quadratic residues
6. Euler Totient Functions
7. Arithmetic Functions
8. Legendre Symbol and Jacobi
9. Primitive roots
10. Partitions

THIRD SEMESTER

1. FUNCTIONAL ANALYSIS - MT9118

(4 hours per week)

Normed linear spaces: Norm on a linear space: Example of norm linear spaces. Semi-norms and quotient spaces. Measurable functions and L^p spaces. Inner Product Space. (14 hours)

Banach Spaces: Incomplete norm Linear Space. Completion of norm linear spaces. Properties of Banach Spaces. Schauder Basis and Separability. Heine Borel Theorem and Riesz Lemma. Best approximation theorem and projection theorem. (14 hours)

Operation on Norm Linear Spaces: Bounded Operators. Norm on $\mathcal{B}(X,Y)$. Riesz-representation theorem. Completeness of $\mathcal{B}(X,Y)$. (10 hours)

Hilbert Spaces: Orthonormal Set and Orthonormal Basis. Bessel's Inequality. Fourier Expansion and Parseval's Formula. Riesz-Fischer Theorem (10 hours)

Hahn-Bannach Theorem and its consequences: The extension theorem and its consequences. (6 hours)

Closed Graph Theorem and its consequences: Closed Graph Theorem. Bounded Inverse Theorem. Open Mapping Theorem. (6 hours)

SUGGESTED MARKS DISTRIBUTION:

Paragraph	1	2	3	4	5	6
Marks	25	25	15	15	10	10

TEXT BOOKS:

1) M. T. Nair, Functional Analysis a first course, PHI
2) B. V. Limaye, Functional Analysis (Wiley Eastern).

REFERENCE BOOKS:

1) G. F. Simmons, Introduction to Topology and Modern Analysis (McGraw-Hill International Edition).
2) G. Backman and L. Narici, Functional Analysis (Academic).
3) P.R. Halmos, Finite dimensional vector spaces (Van Nostrand), 1958.
4) E. Kreyszig, Introduction to Functional Analysis with Applications (John Wiley and Sons).

2. CLASSICAL AND CONTINUUM MECHANICS MT9218

(4 hours per week)

Classical Mechanics: Co-ordinate systems. Plane polar, cylindrical and spherical polar co-ordinate systems. Frame of reference. Rotational frame. Coriolis forces. Motion of system of particles. Conservation laws. Constraints and degrees of freedom. Generalized co-ordinates. Lagrange's and Hamilton's formulations. Canonical Transformation. Poisson's brackets.

(20 hours)

Continuum Mechanics: Suffix notation. Coordinate transformations. Cartesian tensors. Basic Properties. Transpose. Symmetric and Skew tensors. Isotropic tensors. Deviatoric Tensors. Gradient, Divergence and Curl in Tensor Calculus. Integral Theorems. (10 hours)

Continuum Hypothesis. Configuration of a continuum. Mass and density. Description of motion. Material and spatial coordinates. Translation. Rotation. Deformation. Motion. Stress. Deformation of a surface element. Deformation of a volume element. Isochoric deformation. Examples. Stretch and Rotation. Decomposition of a deformation. Deformation gradient. Fundamental laws of continuum mechanics. Equations of linear elasticity. Strain tensors. Infinitesimal strain. Compatibility relations. Principal strains. Equation of Fluid Mechnaics. Material and Local time derivatives. Strain-rate tensor. Transport formulas. Stream lines. Path lines. Vorticity and Circulation. Examples. Stress components and Stress tensor. Normal and shear stresses. Principal stresses. Fundamental basic physical laws. Law of conservation of mass. Principle of linear and momentum. Balance of energy. Examples. Equations of fluid mechanics. Viscous and non-viscous fluids. Stress tensor for a viscous fluid. Navier-Stokes equation. Simple consequences and simple applications.

(30 hours)

SUGGESTED MARKS DISTRIBUTION:

Paragraph	1	2	3
Marks	35	15	50

TEXT BOOKS:

1) Classical Mechanics by H.Goldstein, Narosa Publishing Home,, New Delhi.
2) Classical Dynamics of Particles and Systems by Marion and Thomtron, Third Edition, Horoloma Book Jovanovich College Publisher.
3) Classical Mechanics by P.V.Panat, Narosa Publishing Home,, New Delhi.
4) Classical Mechanics by N.C.Rana and P.S.Joag, Tata Mc-Graw Hill Publishing Company Limited, New Delhi.
5) Introduction to Classical Mechanics by R.G.Takawale and P.S.Puranik, Tata Mc-Graw Hill Publishing Company Limited, New Delhi.
6) Classical Mechanics by J.C.Upadhyaya, Himalaya Publishing House

REFERENCE BOOKS:

1) Continuum Mechanics – D S Chandrashekharaiah and Lokenath Denath
2) Fluid Mechanics – R K Bansal
3) Continuum Mechanics, P Chadwick, Allen and Unwin, 1976.
4) Introduction to the Mechanics of a Continuous Media, L.E. Malvern, Prentice Hall, 1969.
5) A First course in Continuum Mechanics, Y.C. Fung, Prentice Hall (2nd edition), 1977.

6) Fluid Mechanics, Pijush K. Kundu, Ira M. Cohen and David R. Dowling, Fifth Edition , 2010.
7) Fluid Mechanics, C.S.Yih, McGraw-Hill, 1969.
8) Introduction to Fluid Dynamics – G K Batchelor, McGraw – Hill, 1967.

3. ELECTIVE - I

3(a) COMMUTATIVE ALGEBRA - MTDE9318

(4 hours per week)

Recap of Rings and Modules. Nilpotent Elements. Nilradical and Jacobson Ideal. Operation on ideals. Extension and Contraction. (5 hours)

Localisation of Rings and Modules. (10 hours)

Primary Decomposition with demonstration using Singular/Macaulay (10 hours)

Integral Extensions. Going-up, Lying-over and Going-down Theorems. Hilbert's Nullstellensatz, Noether's Normalisation Lemma. (10 hrs)

Noetherian Rings and Modules. Artinian Rings. (15 hours)

Discrete Valuation ring and Dedekind's Domain. (10 hours)

SUGGESTED MARKS DISTRIBUTION:

Paragraph	1+2	3	4	5	6
Marks	20	20	20	20	20

TEXT BOOKS:

1) M. F. Atiyah and I. G. Macdonald, Introduction to Commutative Algebra (Addision-Wesley), 1969.
2) N. S. Gopalakrishnan, Commutative Algebra (Oxonian Press), 1984.

REFERENCE BOOKS:

1) David Eisenbud. Commutative Algebra with a view towards Algebraic Geometry, Springer, GTM.

3(b) MATHEMATICAL METHODS - MTDE9418

(4 hours per week)

Integral Equations: Definition. Volterra and Fredholm integral equations. Solution by separable kernel, Neumann's series resolvent kernel and transform methods. Convergence for Fredholm and Volterra types. Reduction of IVPs, BVPs and eigenvalue problems to integral equations. Hilbert Schmidt theorem. Raleigh Ritz and Galerkin methods. (16 hours)

Asymptotic expansions: Asymptotic expansion of functions. Power series as asymptotic series. Asymptotic forms for large and small variables. Uniqueness properties and Operations. Asymptotic expansions of integrals. Method of integration by parts (include examples where the method fails). Laplace's method and Watson's lemma. Method of stationary phase and steepest descent. (14 hours)

Runge Kutta methods of first, second and fourth order and their stability (Convergence and Truncation error for the above methods). Multistep method. The Adam-Bashfort. Moulton method of second and fourth orders. Boundary- Value problems-Second order finite difference methods. (12 hours)

Numerical solution of Partial differential equations: Difference methods for Elliptic partial differential equations. Difference schemes for Laplace and Poisson's equations. Iterative methods of solution by Jacobi and Gauss Siedel methods. Solution techniques for rectangular and quadrilateral regions. Difference methods for parabolic equations in one-dimension. Methods of Schmidt and Crank-Nicolson. Stability and convergence analysis for Schmidt and Crank-Nicolson methods. A.D.I. method for two-dimensional parabolic equation. Explicit finite difference schemes for hyperbolic equations. Wave equation in one-dimension. (18 hours)

SUGGESTED MARKS DISTRIBUTION:

Chapter	1	2	3
Marks	25	40	35

TEXT BOOKS:

1) I.N. Sneddon, The use of Integral Transforms (Tata Mc Graw Hill, Publishing Company Ltd, New Delhi), 1974.
2) R.P. Kanwal, Linear integral equations theory and techniques (Academic Press, New York), 1971.
3) C.M. Bender and S.A. Orszag, Advanced mathematical methods for scientists and engineers (McGraw Hill, New York), 1978.

4. <u>ELECTIVE - II</u>

4(a) <u>ALGEBRAIC TOPOLOGY - MTDE9518</u>

(4 hours per week)

The Fundamental Group: Recap of continuity of maps between topological spaces, homeomorphisms, connectedness and path-connectedness, compactness, quotient topology. (2 hours)

Homotopy of paths. The fundamental group. Covering spaces. The fundamental group of the circle. (20 hours)

Retractions and fixed points. Applications of Algebraic topology techniques to prove The Brouwer Fixed Point Theorem and The Fundamental Theorem of Algebra. The Borsuk-Ulam Theorem and applications. (10 hours)

Deformation retracts and homotopy type. The fundamental group of the n-dimensional sphere, the real projective space and figure eight space (with full description of how to construct these spaces). (10 hours)

Classification of covering spaces: Equivalence of covering spaces. The universal cover. Deck-transformations (aka covering transformations). Existence of covering spaces. (10 hours)

The Seifert-Van Kampen Theorem: Direct sums of abelian groups. Free product of groups. Free groups. The Seifert-Van Kampen Theorem. The fundamental group of a wedge of circles. (8 hours)

SUGGESTED MARKS DISTRIBUTION:

Chapter	1	2	3
Marks	80	10	10

TEXT BOOKS:

1) I M Singer and John A Thorpe, Lecture notes on elementary topology and geometry.
2) James Munkres, Topology (Pearson Education India, second edition), 2007.

REFERENCE BOOKS:

1) M. A. Armstrong, Basic Topology (Springer Edition).
2) William S Massey, Algebraic Topology: An introduction (Springer).
3) Allen Hatcher, Algebraic Topology (Cambridge University Press).

4(b) OPTIMIZATION TECHNIQUES - MTDE9618

(4 hours per week)

Linear programming problems. Model formulation and graphical solution. Various types of solutions. Simplex method of solving linear programming. Artificial variable techniques. Big M method. Principle of duality. (10 hours)

Transportation problem. Initial Basic Feasible Solution. North-West Corner Rule. Vogel's Approximation Method. MODI method of finding optimal solutions. Assignment problem. Sequencing problem. 'n' jobs two machines problem. 'n' jobs 'm' machines problem. (10 hours)

Replacement problem. Replacement of policy when value of money changes/does not change with time. Replacement of equipment that fails suddenly. Game theory. Two-person zero sum games. Pure and Mixed strategies. Games with saddle point. Principle of dominance. Graphical method. (16 hours)

Decision analysis. Components of decision making. Decision making without probabilities. Maximin–minimax regret. Hurwicz and equal likelihood criterion. Decision making with probabilities. Expected value. Expected opportunity loss criterion. (8 hours)

Network flow models. Shortest route problem. Project management. The CPM and

PERT Networks. Sensitivity analysis. (8 hours)

Waiting Line Theory or Queuing Model. Queuing system or process. Service Mechanism. Steady, Transient and Explosive states in a queue system. Queue Models. (8 hours)

SUGGESTED MARKS DISTRIBUTION:

Paragraph	1	2	3	4	5	10
Marks	20	20	30	15	15	10

TEXT BOOKS:

1) Hamdy A. Taha, Operations Research - An Introduction (8/e, Prentice Hall of India Private Ltd, New Delhi), 2006.
2) Sharma, J.K, Operations Research, Theory and applications (Macmillan), 1997.

REFERENCE BOOKS:

1) Hillier F S and Libermann G J, Introduction to Operations Research (7th Edition, McGraw Hill), 2002.
2) Kanti Swarup, Manmohan and Gupta P. K., Operations Research, Sultan Chand and Sons, New Delhi), 1985.

5. OPEN ELECTIVE
MAKING THE RIGHT DECISIONS - MTOE9718
(2 hours per week)

OBJECTIVE: To enhance the ability to make decisions accurately and with confidence by providing with principles of successful decision making.

INTRODUCTION: You're what you decide and there's a really good way to make these decisions. The principles of decision theory are widely used by major organizations in making strategic decisions. They also apply to all decisions - whether they involve one's finances, one's health or one's relationships.

Note. It isn't necessary to understand Mathematics to learn the principles of Decision Theory.

METHODOLOGY: 'Learn By Doing'

You'll be provided with the opportunity to make decisions. A lot of them. There are Three Important Areas in life in which one can make decisions:

- Decisions About Personal Relationships
- Decisions About What You Should Do With Your Life
- Decisions About Organisations In Which You Participate

WHAT WILL YOU LEARN?

SYLLABUS: Choices. Determining Payoffs (What is in it for me?). Types of decision.

Four major decision criteria: Admissibility criterion. Worst case scenarios. The long term perspective. Make hay while the sun shines (Maximax criterion). Guidelines in deciding which of the four criteria to use. Acquisition and use of information. Decision involving other parties. Cooperative solutions. Vindictive Solutions. (30 hours)

TEXT BOOKS:

1) Miller James -Game theory of work-McGraw Hill-2003
2) Bradley Richard -Decision theory-A formal philosophical-2014
3) North D.Warner -A Tutorial introduction to decision theory-IEEE Transaction on system science and cybernatics, vol ssc.so.3, section-1968
4) Hansson Sven Ove -Decision Theory, A Brief Introduction-Royal Institution of technology-2005
5) Burger Starbird-The Heart of Mathematics-John Wiley and sons,Newyork-2010

6. INTRODUCTION TO MATHEMATICAL RESEARCH - MT9R1

(2 hours per week)

SYLLABUS:

Research Methodology: Introduction to research and research methodology. Scientific methods. Choice of research problem. Literature survey and statement of research problem. Reporting of results. Roles and responsibilities of research student and guide. (10 hours)

Mathematical research methodology: Introducing mathematics Journals. Reading a Journal article. Mathematics writing skills. Standard Notations and Symbols. Using Symbols and Words. Organizing a paper. Defining variables, symbols and notations, Different Citation Styles. IEEE Referencing Style in detail. (10 hours)

Independent Research: The student will begin the study of a research paper(s) and will make presentations/working seminars to the supervisor(s), A project proposal will be the final presentation. (10 hours)

METHOD OF EVALUATION:

The continuous assessment is based on the presentations and seminars during the course of the semester. (20/50 marks). The end semester evaluation is based on the project proposal submitted by each student. (30/50 marks).

FOURTH SEMESTER

1. MEASURE AND INTEGRATION - MT0118

(4 hours per week)

Algebra of sets, sigma algebras, open subsets of the real line. F_σ and G_δ sets, Borel sets, Outer measure of a subset of $\mathbb{R}$. Lebesgue outer measure of a subset of R. Existence, non-negativity and monotonicity of Lebesgue outer measure. Relation between Lebesgue outer measure and length of an interval. Countable sub-additivity of Lebesgue outer measure; translation invariance. (10 hours)

(Lebesgue) measurable sets, (Lebesgue) measure. Complement, union, intersection and difference of measurable sets. Denumerable union and intersection of measurable sets. Countable additivity of measure. The class of measurable sets as a algebra. The measure of the intersection of a decreasing sequence of measurable sets. (9 hours)

Measurable functions. Scalar multiple, sum, difference and product of measurable functions. Measurability of a continuous function and measurability of a continuous image of measurable function. Pointwise convergence and convergence in measures of a sequence of measurable functions. (8 hours)

Lebesgue Integral. Characteristic function of a set. Simple function. Lebesgue integral of a simple function. Lebesgue integral of a bounded measurable function. Lebesgue integral and Riemann integral of a bounded function defined on a closed interval. Lebesgue integral of a non-negative function. Lebesgue integral of a measurable function. Properties of Lebesgue integral. (7 hours)

Convergence Theorems and Lebesgue integral. The bounded convergence theorem. Fatou's Lemma. Monotone convergence theorem. Lebesgue convergence theorem. (7 hours)

Differentiation of Monotone functions. Vitali covering lemma. Functions of Bounded variation. Differentiability of an integral. Absolute continuity and indefinite integrals. (9 hours)

L_p spaces. Holder and Minkowski inequalities. Convergence and completeness. Riesz-Fischer Theorem. Bounded linear functionals. Riesz representation theorem and illustrative examples. Measure spaces and Signed measures. The Radon Nikodyn theorem. (9 hours)

SUGGESTED MARKS DISTRIBUTION:

Paragraph	1	2	3	4	5	6	7
Marks	10	10	15	15	20	20	10

TEXT BOOKS:

1) H.L. Royden : Real Analysis, Macmillan, 1963

REFERENCE BOOKS:

1) P.R. Halmos : Measure Theory, East West Press, 1962

2) W. Rudin : Real Complex Analysis, McGraw Hill , 1966

2. ELECTIVES

2(a) GRAPH THEORY - MTDE0218

(4 hours per week)

Connectivity: Cut-vertex, Bridge(Theorems). Blocks and Block Graph. Vertex-connectivity. Edge-connectivity. Connectivity pair of a graph and some external problems. Mengers Theorems, Properties of n-connected graphs with respect to vertices and edges. (12 hours)

Colorability: Vertex Coloring, Color class. n-coloring. Chromatic index of a graph. Chromatic number of standard graphs. Bi-chromatic graphs. Colorings in critical graphs. Relation between chromatic number and clique number/independence number/maximum degree. Edge coloring. Edge chromatic number of standard graphs. Coloring of a plane map. Four color problem. Five color theorem. Uniquely colorable graph. Chromatic polynomial. (12 hours)

Matchings and factorization: Matching- perfect matching. Augmenting paths. Maximum matching. Hall's theorem for bipartite graphs. The personnel assignment problem. A matching algorithm for bipartite graphs. Factorizations - 1-factorization, 2-factorization. Partitions-degree sequence. Havel's and Hakimi algorithms and graphical related problems. (12 hours)

Directed Graphs: Preliminaries of digraph. Oriented graph. In-degree and out-degree. Elementary theorems in digraph. Types of digraph. Tournament. Cyclic and transitive tournament. Spanning path in a tournament. Tournament with a Hamiltonian path. Strongly connected tournaments. Problems on Tournament. (12 hours)

Domination concepts and other variants: Dominating sets in graphs, domination number of standard graphs, minimal dominating set. Bounds of domination number in terms of size, order, degree, diameter. Covering and independence number. Domatic number, Domatic number of standard graphs. (9 hours)

Max flow. Min cut theorem and related problems. (3 hours)

SUGGESTED MARKS DISTRIBUTION:

Paragraph	1	2	3	4	5	6	7
Marks	10	10	15	15	20	20	10

TEXT BOOKS:

1) F. Harary: Graph Theory, Addison -Wesley,1969

2) G.Chartrand and Ping Zhang: Introduction to Graph Theory. McGrawHill, International edition (2005)

3) J.A.Bondy and V.S.R.Murthy: Graph Theory with Applications, Macmillan, London, (2004).

REFERENCE BOOKS:

1) D.B.West, Introduction to Graph Theory,Pearson Education Asia, 2nd Edition, 2002.
2) Charatrand and L. Lesnaik-Foster: Graph and Digraphs, CRC Press (Third Edition), 2010.
3) T.W. Haynes, S.T. Hedetneime and P. J. Slater: Fundamental of domination in graphs, Marcel Dekker. Inc. New York.1998.
4) J. Gross and J. Yellen: Graph Theory and its application, CRC Press LLC, Boca Raton, Florida, 2000.
5) Norman Biggs: Algebraic Graph Theory, Cambridge University Press (2nd Ed.)1996.
6) Godsil and Royle: Algebraic Graph Theory: Springer Verlag, 2002.
7) N. Deo: Graph Theory: Prentice Hall of India Pvt. Ltd. New Delhi – 1990

2(b) **NON-LINEAR DYNAMICS AND CHAOS THEORY - MTDE0318**

(4 hours per week)

History of Dynamics. Importance of being nonlinear. A Dynamical view of the world. (3 hours)

One-Dimensional flows Flows on the line. A Geometric way of thinking. Fixed Points and stability. Population growth. Linear stability analysis. Existence and uniqueness. Impossibility of oscillations. Potentials. Solving equations on the computer (demo). (7 hours)

Bifurcations. Saddle node. Transcritical. Pitchfork bifurcations. Laser threshold. Imperfect bifurcations. (6 hours)

Flows on the circle. Uniform and nonuniform oscillators. Overdamped pendulums. Superconducting Josephson junction. (6 hours)

Two - dimensional flows. Linear Systems. Definitions and examples. Classifications of linear systems (3 hours)

Phase plane. Phase portraits. Rabbit and sheep. Conservative systems. Reversible system. Pendulum. Index theory (6 hours)

Limit Cycles. Ruling out closed orbits. Poincare - Bendexsion theorem. Leinard systems. Relaxation oscillators. Weakly nonlinear oscillators. (5 hours)

Bifurcations and Poincare maps. Pitchfork, Hopf bifurcations. Coupled oscillators. Quasiperiodicity. Poincare maps (6 hours)

Chaos: Lorenz Equation. A chaotic water wheel. Simple properties of Lorentz equation. Chaos on a strange attractor. Lorenz maps (5 hours)

One D maps. Fixed points and cobwebs. Logistic maps. Lyapunov exponents (6 hours)

Fractals. Countable and uncountable sets. Cantor set. Dimension of self similar fractals. Box dimensions. Correlation dimension. (3 hours)

Strange Attractor - the simplest examples. Henon maps. Rossler system(demo) (4 hours)

SUGGESTED MARKS DISTRIBUTION:

Paragraph	1	2	3	4	5	6	7	8	9	10
Marks	10	10	10	10	10	10	10	10	10	10

TEXT BOOKS:

1) Nonlinear Dynamics and Chaos, with applications to Physics, Biology, Chemistry and Engineering – Steven H Strogatz – perseus book – 1994
2) Chaos – an introduction to dynamical systems – Aligood KT, Sauer and Yorke – springer - 1996

REFERENCE BOOKS:

1) C Sprott, Chaos and time series analysis, Cambridge University Press

2(c) FLUID DYNAMICS - MTDE0418

(4 hours per week)

One, Two and Three Dimensional Invisid Incompressible Flow: Bernoulli equation. Applications of Bernoulli equation. Circulation theorems. Circulation concept. Kelvin's theorem. Constancy of circulation. Laplace equations. Stream functions in two and three dimensional motion. Two dimensional flow: Rectilinear flow. Source and sink. The theorem of Blasius. (15hours)

Two Dimensional flows of Inviscid fluid: Flow between parallel flat plates. Couette flow. Plane Poiseuille flow. The Hagen-Poiseuille flow. Flow between two concentric rotating cylinders. (10hours)

Dimensional Analysis and Similarity: Introduction to heat transfer. Different modes of heat transfer - conduction, convection and radiation. Steady and unsteady heat transfer. Free and forced convection. Non-dimensional parameters determined from differential equations. Buckingham's Pi Theorem. Non-dimensionalization of the Basic Equations. Non-dimensional parameters and dynamic similarity. (15hours)

Heat Transfer and Thermal Instability. Shear Instability: Stability of flow between parallel shear flows. Squire's theorem for viscous and inviscid theory. Rayleigh stability equation. Derivation of Orr-Sommerfeld equation assuming that the basic flow is strictly parallel. Basic concepts of stability theory. Linear and Non-linear theories. Rayleigh Benard Problem. Analysis into normal modes. Principle of Exchange of stabilities. First variation principle. Different boundary conditions on velocity and temperature.

(20hours)

SUGGESTED MARKS DISTRIBUTION:

Chapter	1	2	3
Marks	35	15	50

TEXT BOOKS:

1) S. W. Yuan, Foundations of fluid mechanics, Prentice Hall of India, 2001.
2) M. D. Raisinghania, Fluid Dynamics, S. Chand and Company Ltd., 2010.
3) Drazin and Reid, Hydrodynamic instability, Cambridge University Press, 2006.
4) S. Chardrasekhar, Hydrodynamic and hydrodmagnetic stability, Oxford University Press, 2007.

REFERENCE BOOKS:

1) D. J. Tritton, Physical fluid Dynamics, Clarendon Press, 1988.
2) P. K. Kundu, Ira M. Cohen and David R Dowling, Fluid Mechanics, 5th ed., Academic Press, 2011.
3) F. M White, Fluid Mechanics, Tata Mcgraw Hill. 2011.
4) D. S. Chandrasekharaiah and L. Debnath, Continuum mechanics, Academic Press, 1994 (Reprint).
5) P. K. Kundu, Ira M. Cohen and David R. Dowling, Fluid Mechanics, Fifth Edition, 2010.
6) G.K. Batchelor, An introduction to fluid mechanics, New Delhi: Foundation Books, 1984.
7) F. Chorlton, Text book of fluid dynamics, New Delhi: CBS Publishers Distributors, 1985.
8) J.F. Wendt, J.D. Anderson, G. Degrez and E. Dick, Computational fluid dynamics: An introduction, Springer-Verlag, 1996.
9) F. M White, Fluid Mechanics, Tata Mcgraw Hill. 2010.

2(d) **ALGEBRAIC GEOMETRY - MTDE0518**

(4 hours per week)

Affine Varieties: Polynomial ring in n variables. Affine n-space. Algebraic sets. Hilbert basis theorem. The Zariski topology. Ideal of a set. Radical of an ideal. Hilbert's Nullstellensatz. One-One correspondence between (i) algebraic sets and radical ideals (ii) points and maximal ideals. Noetherianness, Irreducibility and Dimension.

(15 hours)

Functions, Morphisms and Varieties: Functions on affine varieties. Co-ordinate ring of an affine variety. Field of rational functions. Local ring. Ring of regular functions. Distinguished open sets. Morphisms between affine varieties. Correspondence between morphisms and k-algebra homomorphisms. (15 hours)

Projective Varieties: Projective n-space. Homogeneous co-ordinates and homogeneous ideals. Projective algebraic sets. Projective varieties. Irrelevant ideal. Cone over projective algebraic sets. The projective Nullstellensatz. Homogeneous co-ordinate ring. Field of rational functions. Affine open cover of projective varieties. One-one correspondence between projective algebraic sets and radical homogeneous ideals. (15 hours)

The Local Ring of Germs of Functions at a Point. The Function Field of Functions on Large Open Sets. Rings of Functions on Projective Varieties. Regular or Smooth Points and Manifold Varieties or Smooth Varieties. (15 hours)

SUGGESTED MARKS DISTRIBUTION:

Chapter	1	2	3	4
Marks	20	20	20	20

TEXT BOOKS:

1) Robin Harthsrone, Algebraic Geometry, GTM Springer
2) Andreas Gathman. Algebraic Geometry (Lecture notes)

REFERENCE BOOKS:

1) Igor Shafarevich, Basic Algebraic Geometry-I, Springer Verlag
2) C Musili, Algebraic Geometry for beginners, Hindustan Book Agency, 2001.
3) K Smith, P Kekalainen, W Traves, L Kahanpaa, An Invitation to Algebraic Geometry, Universitext.

2(e) <u>REPRESENTATION THEORY OF FINITE GROUPS - MTDE0618</u>

(4 hours per week)

Group Representations : Definition and first examples of representations, Equivalence of representations, Invariant subspace, Direct sums of representations, Irreducible representation, Unitary representations, Complete reducibility (Maschke's theorem). (10 hours)

Character Theory and the Orthogonality Relations : Morphisms of representations, Schur's lemma, Orthogonality relations, Characters and class functions, The regular representation, Representations of abelian groups. (8 hours)

Fourier Analysis on Finite Groups: Periodic Functions on Cyclic Groups. The Convolution Product. Fourier Analysis on Finite Abelian Groups. (8 hours)

Burnside's Theorem: Any non-abelian group of order $p^a q^b$ is simple. (8 hours)

Group Actions and Permutation Representations: Group Actions. Permutation Representations. Fixed Subspace. Burnside's Lemma (8 hours)

Induced Representations: Induced Characters and Frobenius Reciprocity. Induced Representations. Mackey's Irreducibility Criterion . (8 hours)

Representation Theory of the Symmetric Group: Partitions and Tableaux. Constructing the Irreducible Representations. (10 hours)

SUGGESTED MARKS DISTRIBUTION:

Chapter	1	2	3	4	5	6	7
Marks	15	15	15	10	15	15	15

TEXT BOOKS:

1) Benjamin Steinberg. Representation Theory of Finite Groups, an introductory approach, Springer International Edition.
2) J P serre. Linear Representations of Finite Groups, Springer.

REFERENCE BOOKS:

1) Martin Liebeck and Gordon James. Characters of Groups, Cambridge press, second edition.
2) W Fulton and J Harris. Representation Theory: A first course, GTM, corrected edition

2(f) <u>BIO-STATISTICS - MTDE0718</u>

(4 hours per week)

Advanced Distribution theory: Discrete distributions: Uniform, Binomial, Poisson, Geometric, Power series. Continuous distributions: Uniform, Normal, Exponential, Gamma, Chi-square, t, F, Lognormal. Additive properties. Lack of memory properties. Limit theorems: Law of large numbers and central limit theorem. Chebychev Inequality. Jensen Inequality. Markov Inequality. (10 hours)

Estimation Theory: Point estimation-unbiasedness-consistenc. Uniformly minimum variance unbiased estimator. Necessary and sufficient condition for an estimator to be an UMVUE. Properties of UMVUE, Examples. Method of maximum likelihood. Properties of ML Estimators. Consistent asymptotic normal (CAN) estimators, examples. Invariance property of CAN estimators. Resampling methods-Jackknife method of estimation, Bootstrap's method. Equivariant estimation. Minimum risk equivariant estimator for location models, scale models and location-scale models. Illustrative examples. (12 hours)

Multivariate Analysis: Multivariate continuous distributions: Bivariate normal, exponential (Marshall and Olkin) distributions. Properties. Multivariate extensions of Convex loss function. Rao-Blackwell theorem. Lehmann-Scheffe theorem, examples. Cramer-Rao inequality for the multiparameter case. Multinomial and Multivariate Poisson distributions. Covariance and conditional expectations and special distributions.

(6 hours)

Stochastic Processes: Elements of Stochastic processes, simple examples. Classification of general stochastic processes. Stationary independent increment process. Properties. Markov Chains – discrete in time. Examples. Classification of states of a Markov Chain. Recurrence. Basic limit theorem of Markov Chains. Absorption probabilities. Criteria for recurrence. Markov Chains continuous in time. Examples. General Pure birth process, Poisson process, Birth – Death process. Finite state continuous time Markov Chains. (12 hours)

Design of Experiments: Review of Linear models – Block Design, C-matrix and its properties. Analysis of block design. (CRD) completely Randomized design. (RBD)-Randomized Block Design. (LSD)- Latin Square Design. (RLSD) - Repeated Latin Square Design. Missing plot techniques. Factorial Design – 2n; 3n factorial designs. Partial confounding and complete confounding. Confounding in more than two blocks.

(10 hours)

Statistical Genomics: Introduction to Biology. Central dogma: DNA, RNA, proteins, traits. Recent massive high-throughput technologies. Statistical issues commonly encountered in genomics and genetics. Biological sequence analysis. DNA sequence analysis. Protein sequence analysis. Hidden Markov Models (HMM) (10 hours)

SUGGESTED MARKS DISTRIBUTION:

Chapter	1+2	3+4	5+6
Marks	35	30	35

REFERENCE BOOKS:

1) Cox, D.R. and Miller, H.D. (1965). Theory of Stochastic Processes – Chapman and Hall, London 3rd edition.
2) Feller, W (1972). An introduction to probability theory and its applications, Vol.I, Wiley Eastern Ltd.
3) Chakrabarthi, M.C. (1970). Mathematics of design and analysis of experiments. Asia Publishing House.
4) Das, M.N. and Giri, N. (1988). Design and Analysis of Experiments, Wiley Eastern.
5) Ferderer,W.T. (1993). Experimental Designs – Theory and Applications, McMillan.
6) Joshi,D.D (1987). Linear estimation and design of experiments. Wiley Eastern.
7) Johnson,R.A. and Wichern, D.W. Prentice (1988): Applied Multivariate Statistical Analysis. Prentice Hall International, Inc.
8) Cramer H.(1946). Mathematical methods of Statistics. Princeton University Press, N.J.
9) Ferguson,T.S(1967). Mathematical Statistics – A decision theoretic approach.Academic Press.

10) Kendall, M.G and Stuart,A.(1967).The Advanced Theory of Statistics. Vol.2. Inference and Relationship. Hafner Publishing Co., New York.
11) Lehmann,E.L.and Casella,G.(1998).Theory of point estimation. Springer-Verlag.
12) Rao,C.R.(1973).Linear Statistical inference and its applications. Wiley Eastern Ltd.
13) Zacks, S. (1971). The theory of Statistical inference. John Wiley and Sons, N.Y.
14) Prem Narain. Statistical Genetics (1990).

2(g) NUMERICAL ANALYSIS - MTDE0818

(4 hours per week)

Unit 1: Introduction: Error, Types of error: Absolute, Relative and Percentage error, Round off error Methods of computation: Accuracy, Operation count, Stability, loss of significant digits and convergence.

Unit 2: System of scalar equations

2a: Canonical forms and Operator form of Linear systems. Norms and Normed spaces, norm of a linear operator, Conditioning of a linear system, Condition number and characterization of a linear system by means of its condition number.

2b: Standard Gaussian elimination, Tridiagonal elimination, Cyclic tri-diagonal elimination, LU factorization, Dolittle, Crouts and Cholesky factorization.

2c: Iterative methods for solving linear systems: Richardson method and Successive over Relaxation (SOR) method. Finite difference Dirichlet problem for Poisson equation

Unit 3: Curve fitting: Least square regressions, linear regressions, polynomial regressions, multiple linear regressions and non-linear regression.

Unit 4: Interpolation: Newtons divided difference, Lagrange's, inverse Lagrange's, Spline – Linear, Quadratic and cubic splines interpolation, Multi–dimensional interpolation and Hermite interpolation (including derivations and error analysis for all the methods).

Unit 5: Differentiation and Integration and ODE: Newton – Cotes integration formulae, Romberg integration, Gauss quadrature, Richardson extrapolation. Shooting method.

SUGGESTED MARKS DISTRIBUTION:

Unit	1	2	3	4	5
Marks					

TEXT BOOKS:

1) M.K. Jain: Numerical solution of differential equations, Wiley Eastern (1979), Second Edition.
2) C.F. Gerald and P.O. Wheatley : Applied Numerical Methods, Low- priced edition, Pearson Education Asia (2002), Sixth Edition.

3) D.V. Griffiths and I.M. Smith, Numerical Methods for Engineers, Blackwell Scientific Publications (1991).

REFERENCE BOOKS:

1) S.C. Chapra, and P.C. Raymond : Numerical Methods for Engineers, Tata Mc Graw Hill, New Delhi (2000)

2) R.L. Burden, and J. Douglas Faires : Numerical Analysis, P.W.S. Kent Publishing Company, Boston (1989), Fourth edition.

3) S.S. Sastry : Introductory methods of Numerical analysis, Prentice- Hall of India, New Delhi (1998).

4) M.K. Jain, S.R.K. Iyengar and R.K. Jain : Numerical methods for scientific and Engineering computation, Wiley Eastern (1993)

5) R.V. Churchill: Operational Mathematics, Mc. Graw Hill, New York, 1958

2(h) BASIC OPERATOR THEORY - MTDE0918

(4 hours per week)

Dual Spaces: Dual of $l^p(n), C[a,b], L^p[a,b]$, Reflexivity and weak convergence, Best approximation in reflexive spaces. (15 hours)

Spectral Results for Banach Space Operators: Eigen spectrum and approximate Eigenspectrum. Spectral radius and spectral mapping theorem (10 hours)

Operator on Hilbert Spaces: Adjoint of an operator, Compactness of adjoint operator, adjoint of an unbounded operator, Self-adjoint, normal, unitary operators, Numerical Range and Numerical Radius (15 hours)

Compact Operators: Compact Linear maps, Spectrum of a Compact Operator (10 hours)

Compact Self-adjoint operators: Spectral Representation of compact self adjoint operators. (10 hours)

SUGGESTED MARKS DISTRIBUTION:

Paragraph	1	2	3	4	5
Marks	25	20	25	15	15

TEXT BOOKS:

1) T. Nair: Functional Analysis: A First Course, Wiley Eastern, 1981.

2) B.V. Limaye: Functional Analysis, Second Edition, New Age International, 1996.

REFERENCE BOOKS:

1) G. F. Simmons, Introduction to Topology and Modern Analysis (McGraw-Hill International Edition).

2) G. Backman and L. Narici, Functional Analysis (Academic).

3) P.R. Halmos, Finite dimensional vector spaces (Van Nostrand), 1958.

4) E. Kreyszig, Introduction to Functional Analysis with Applications (John Wiley and Sons).

2(i) DIFFERENTIAL GEOMETRY MTDE01018

(4 hours per week)

Calculus on Euclidean Space: Euclidean space. Natural coordinate functions. Differentiable functions. Tangent vectors and tangent spaces. Vector fields. Directional derivatives and their properties. Curves in E3. Velocity and speed of a curve. Reparametrization of a curve. 1-forms and Differential forms. Wedge product of forms. Mappings of Euclidean spaces. Derivative map. (15 hours)

Frame Fields: Arc length parametrization of curves. Vector field along a curve. Tangent vector field, Normal vector field and Binormal vector field. Curvature and torsion of a curve. The Frenet formulas Frenet approximation of unit speed curve and Geometrical interpretation. Properties of plane curves and spherical curves. Arbitrary speed curves. Cylindrical helix Covariant derivatives and covariant differentials. Cylindrical and spherical frame fields. Connection forms. Attitude matrix. Structural equations. Isometries of E3- Translation, Rotation and Orthogonal transformation. The derivative map of an isometry. (15 hours)

Calculus on a Surface: Coordinate patch. Monge patch. Surface in E3 . Special surfaces- sphere, cylinder and surface of revolution. Parameter curves, velocity vectors of parameter curves, Patch computation. Parametrization of surfaces- cylinder, surface of revolution and torus. Tangent vectors, vector fields and curves on a surface in E3. Directional derivative of a function on a surface of E3. Differential forms and exterior derivative of forms on surface of E3. Pull back functions on surfaces of E3. (15 hours)

Shape Operators: Definition of shape operator. Shape operators of sphere, plane, cylinder and saddle surface. Normal curvature, Normal section. Principal curvature and principal direction. Umbilic points of a surface in E3. Euler's formula for normal curvature of a surface in E3. Gaussian curvature, Mean curvature and Computational techniques for these curvatures. Minimal surfaces. Special curves in a surface of E3 -Principal curve, geodesic curve and asymptotic curves. Special surface - Surface of revolution. (15 hours)

SUGGESTED MARKS DISTRIBUTION:

Chapter	1	2	3	4
Marks	20	20	20	20

TEXT BOOKS:

1) Barrett O' Neil : Elementary Differential Geometry. Academic Press, New York and London, 1966
2) T.J.Willmore : An introduction to Differential Geometry. Clarendon Press, Oxford 1959.

REFERENCE BOOKS:

1) D.J.Struik : Lectures on Classical Differential Geometry, Addison Wesley, Reading, Massachusetts, 1961.
2) Nirmala Prakassh: Differential Geometry- an integrated approach. Tata McGraw-Hill, New Delhi, 1981.

2(j) **MATHEMATICAL FINANCE - MTDE01118**

(4 hours per week)

Basics of Financial Markets: Introduction and main theme of mathematical finance, financial markets terminology, time value of money, Bonds and bond's pricing, yield, term structure of interest rates. Financial instruments, types of derivatives, concept of arbitrage. (5 hours)

Portfolio Management: Portfolios, returns and risk, risk-reward analysis, mean variance portfolio optimization. Markowitz model, Capital Asset Pricing Models (CAPM). (10 hours)

Probability Essential: Probability spaces. Filtrations, Random variables, conditional expectations, Random processes. Martingales. (6 hours)

Discrete-time Finance: Pricing by arbitrage. Risk-neutral probability measures. Valuation of contingent claims. Pricing and hedging of European and American derivatives as well as fixed-income derivatives in Cox-Ross-Rubinstein (CRR) model. (12 hours)

Stochastic Calculus: Brownian motion. martingales, Ito's formula, Ito's integral, risk-neutral measure, SDE; Risk-neutral measure, Girsanov's theorem, Martingale representation theorems, Representation of Brownian martingales, Feynman-Kac formula. (12 hours)

Continuous-time Finance: Black-Scholes-Merton (BSM) model, derivation of the BSM partial differential equation. The Black-Scholes formula. Self-financing strategies and model completeness. Risk neutral measures. The fundamental theorems of asset pricing. Pricing of American options. Forwards and futures in BSM model. (15 hours)

SUGGESTED MARKS DISTRIBUTION:

Chapter	1	2	3	4	5	6
Marks	6	14	6	22	22	30

TEXT BOOKS:

1) A. O. Petters and X. Dong, An Introduction to Mathematical Finance with Applications (Springer, New York, 2016)
2) S. Roman, Introduction to the Mathematics of Finance (Springer, New York, 2004)
3) S. Ross, An Elementary Introduction to Mathematical Finance, Third Edition (Cambridge U. Press, Cambridge, 2011)

REFERENCE BOOKS:

1) M. Capinski and T. Zastawniak, Mathematics for Finance: An Introduction to Financial Engineering, Springer, 2005.
2) J. C. Hull, Options, Futures and Other Derivatives, 7th Edition, Pearson Education, 2009.
3) S. Shreve, Stochastic Calculus for Finance, Vol. 1 and Vol. 2, Springer, 2004.

3. PROJECT - MT9R2

Prerequisites: Introduction to research of the third semester.

Goal: The course carries 6 credits. A student will choose a topic (either a research paper or continuation of the third semester topic). The student will pose a research question and investigate under the supervision of a faculty member. In the course of the semester they are required to present the progress of their work. At the end of the course they would write a dissertation.

Grading scheme: The student's performance will be evaluated based on the presentations (working seminars to the supervisor), the dissertation and final presentation and viva-voce as follows:

- Proposal - 20 marks
- Progress Presentation - 10 + 20 marks internals
- Final Presentation - 30 marks (10-guide, 10+10-two externals)
- Thesis - 20 marks